Heat and Mass Transport During Microwave Heating of Mashed Potato in Domestic Oven - Sensitivity Analysis

Jiajia Chen, Krishnamoorthy Pitchai, Sohan Birla, Mehrdad Negahban, David Jones, Jeyam Subbiah

Conference of Food Engineering
April 9, 2014, Omaha, NE
Food quality and safety issues

• Nonuniform heating
• Dryness/Sogginess
• Outbreaks
Microwave model can be used for

- Food product design…

  “Designing food products that respond well to microwave cooking is a mixture of art and science, with heavy emphasis on the science.”

Microwave heating process

- Microwave heating
- Vapor convection
- Internal heating
- Internal water vaporization
- Liquid pumping
- Food sample

- Multiphysics
  - Electromagnetic heating
  - Heat transfer
  - Mass transfer
  - Momentum transfer
Model physics

- Electromagnetic Maxwell’s equations

\[
\nabla \times \mu_r^{-1}(\nabla \times \mathbf{E}) - \left(\frac{2\pi f}{c}\right)^2 (\varepsilon_r - i\varepsilon'')\mathbf{E} = 0
\]

- Energy Conservation

\[
Q_m = \pi f \varepsilon_0 \varepsilon'' \mathbf{E}^2
\]

\[
\frac{\partial}{\partial t} \left( \sum_{i=s,w,g} \rho_i \cdot C_{p,i} \cdot T \right) + \nabla \cdot \left( \sum_{i=w,g} \rho_i \cdot C_{p,i} \cdot \mathbf{u}_{i,\text{eff}} \cdot T \right) = \nabla \cdot (k_{\text{eff}} \cdot \nabla T) - \lambda \cdot \dot{i} + Q_m
\]
Model physics

- **Mass Conservation**
  \[
  \frac{\partial c_i}{\partial t} + \nabla \cdot (-D_i \nabla c_i) + u \cdot \nabla c_i = \frac{\dot{I}}{M_w}
  \]

- **Momentum Conservation**
  \[
  u_i = - \frac{k_{in,i} \cdot k_{r,i}}{\mu_i} \nabla P
  \]

- **Phase Change of Vaporization**
  \[
  \dot{I} = \frac{K M_w (p_{v,eq} - p_v) \Phi S_g}{RT}
  \]

Model parameters

- $\varepsilon_r = \varepsilon' - \varepsilon''$ Dielectric properties
- $\rho$ Density
- $C_p$ Specific heat capacity
- $k$ Thermal conductivity
- $K$ Evaporation rate constant
- $k_{in}$ Intrinsic permeability
- $D$ Diffusion coefficient
Some parameters are not available

- Evaporation rate constant (K)
  - Be related to the speed of heating in a process
  - Higher values \( (K > 100 \text{ s}^{-1}) \) for more intense power sources, such as in frying
  - Lower values \( (K < 1 \text{ s}^{-1}) \) for lower intensity processes, such as baking

Some parameters are not available

- Intrinsic permeability
  - $10^{-14} \sim 10^{-17} \text{ m}^2$ for water
  - $10^{-13} \sim 10^{-17} \text{ m}^2$ for gas
Some parameters are not available

- Diffusion coefficient
  - $10^{-5}$ to $10^{-9}$ m$^2$·s$^{-1}$ for water
  - $10^{-5}$ to $10^{-6}$ m$^2$·s$^{-1}$ for gas
Sensitivity analysis

• Provides insight into which parameters are critical and need to be controlled in the process
• Determines the most significant parameters that need to be measured accurately for reliable model prediction

Objectives

• Perform sensitivity analysis on
  – Evaporation rate constant (K)
  – Intrinsic permeability of water and gas
  – Diffusion coefficient of water and gas
  – 10% and 1000% of baseline model
Baseline values

• Evaporation rate constant (K, l/s)
  – 50 l/s

• Intrinsic permeability (kin, m²)
  – $2 \times 10^{-15}$ m² for water
  – $1 \times 10^{-14}$ m² for gas

• Diffusion coefficient (D, m²·s⁻¹)
  – $10^{-7} \cdot \exp(-2.8 + 2.0M)$ m²·s⁻¹ for water
  – $2.6 \times 10^{-5}$ m²·s⁻¹ for gas
Multiphysics Modeling

- Geometry

- Cavity
- Food
- Turntable
- Magnetron
- Waveguide
- Bump
- Crevice
Multiphysics Modeling

• Meshing

Free tetrahedral elements
Multiphysics Modeling

• Simulation strategy

- Calculate EM field for one location
- Rotate food product to next location
- Average EM fields of 12 locations
- Update material properties
- Calculate heat, mass, and momentum transfer for one rotation time
Experimental validation

Thermal camera

Fiber-optic sensors

Total moisture loss

Locations of fiber-optic sensors
Evaporation rate constant, $K$

10% of baseline $K$ value

1000% of baseline $K$ value
Evaporation rate constant, $K$

![Graph showing evaporation rate constant with different conditions and their corresponding total moisture loss over time.](image-url)
Intrinsic permeability

- Temperature

Intrinsic permeability of water

Intrinsic permeability of gas

1000% of baseline $k_{in,w}$
RMSE = 17.5 °C

10% of baseline $k_{in,w}$
RMSE = 4.4 °C

10% of baseline $k_{in,g}$
RMSE = 1.5 °C

1000% baseline $k_{in,g}$
RMSE = 3.9 °C

$k_{in,w} = 2 \times 10^{-15} \text{m}^2, °C$

$k_{in,g} = 1 \times 10^{-14} \text{m}^2, °C$
Intrinsic permeability

- Total moisture loss

Intrinsic permeability of water

Intrinsic permeability of gas
**Diffusion coefficient**

- **Temperature**

![Graph 1](image1)

- **Diffusion coefficient of water**
  
  \[ D_w = 10^{-7} \cdot \exp(-2.8+2.0M) \text{ m}^2\cdot\text{s}^{-1}, \text{ °C} \]

  - 100% of baseline \( D_w \)
  - RMSE = 0.8 °C
  - 10% of baseline \( D_w \)
  - RMSE = 3.1 °C

![Graph 2](image2)

- **Diffusion coefficient of gas**
  
  \[ D_g = 2.6 \times 10^{-5} \text{ m}^2\cdot\text{s}^{-1}, \text{ °C} \]

  - 10% of baseline \( D_g \)
  - RMSE = 16.1 °C
  - 100% of baseline \( D_g \)
  - RMSE = 28.2 °C
Diffusion coefficient

• Total moisture loss

![Graph showing total moisture loss over time for different conditions](image1)

- **Diffusion coefficient of water**

![Graph showing diffusion coefficient of gas over time for different conditions](image2)

- **Diffusion coefficient of gas**
Sensitivity analysis

10% of baseline parameter value

1000% of baseline parameter value
Conclusions

• The gas diffusion coefficient and intrinsic water permeability were the most sensitive parameters.
• The sensitive parameters need to be determined to develop reliable and robust heat and mass transfer models.
Acknowledgements

- ConAgra Foods, Inc., Omaha.
- USDA CSREES – NIFSI grant (Project number: 2008-51110-04340)
Thank you very much!

Chenjj0422@huskers.unl.edu
Jeyam.subbiah@unl.edu